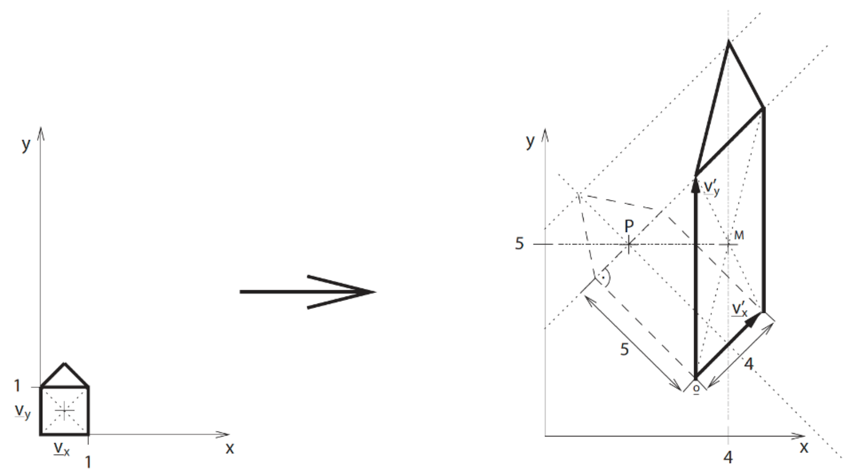
Computer Graphics Worksheet
Transformations and Animation

Dr. Sergey Kosov
Jacobs University Bremen

Problem 1. Transformations

In the picture below the left house should be transformed into the house on the right. The point M is at (4, 5) and lines that look to be parallel are parallel. Please specify the complete transformation matrix as a sequence of primitive transformations (there's no need to calculate the final matrix). Do not guess any numbers.



Problem 2. Homogeneous Coordinates

a) Show that multiplying the homogeneous point $(x, y, z, w \neq 0)$ with an arbitrary scalar $\alpha \neq 0$ yields an equivalent homogeneous point again.
b) Shaw that the component wise addition of three homogenous point $(a_0, b_0, c_0, 1)$, $(a_1, b_1, c_1, 1)$, and $(a_2, b_2, c_2, 1)$ yields the center between that points.

Problem 3. Rotation transform

Show that an arbitrary rotation around the origin in 2D can be represented by a combination of a shearing in y, a scaling in x and y and a shearing in x in this order. You have to derive the shearing and scaling matrices to an arbitrary rotation R:

$$R(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

Problem 4. Affine Spaces

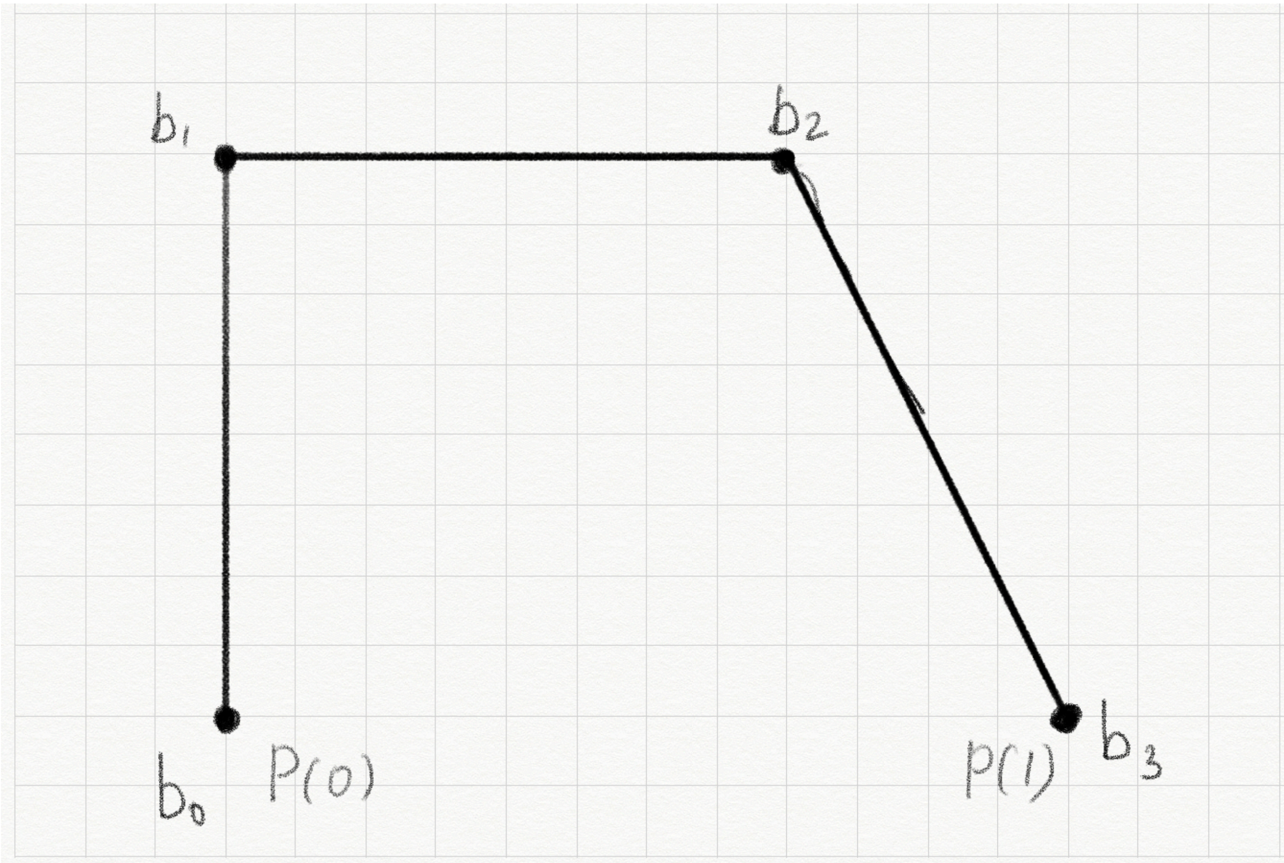
Prove that the set of points $A = \left\{(x, y, z, w) \in \mathbb{R}^4 \,|\, w = 1\right\}$ is an affine space. What is the associated vector space? You do *not* have to show that the associated vector space is a vector space. What is the difference between a point and a vector in that affine space?

Definition of an affine space: An affine space consists of a set of points P, an associated vector space V and an operation $+ \in P \times V \to P$ that fulfils the following axioms:

1. For $p \in P$ and $v, w \in V : (p + v) + w = p + (v + w)$
2. For $p, q \in P$ there exists a unique $v \in V$ such that $p + v = q$

Problem 5. Bézier Curve

Bézier spline $P(t)$ is defined by 4 points: start point b_0, end point b_3 and 2 control points b_1, b_2. Using the DeCasteljau algorithm please draw 3 points (and optionally the supporting lines) in the figure below, which correspond to the values $t_1 = 0.25$, $t_2 = 0.5$ and $t_3 = 0.75$.



Note: value $t = 0$ corresponds to point b_0 and $t = 1$ to point b_3, *i.e.* $P(0) = b_0, P(1) = b_3$.
Hint: This problem should be solved graphically. No formulas are needed.